GONJA
TRADITIONAL AND CULTURAL NUMERALS

MAHAMA IBRAHIM WATARA

THE GONJA TRADITIONAL NUMERALS

3bi Mathematical System of the Gonja People of the Ancient Gonja Kingdom

A Comprehensive Historical, Archaeological, and Cultural Monograph on the Indigenous Mathematical Heritage of the Gonja People

Based on Archaeological Findings from Salaga, Buipe, and Kafaba Excavations of 1970-75 and Related Historical Records

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For Global Cultural Heritage Preservation and UNESCO Recognition

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ABSTRACT

This monograph presents a comprehensive examination of the 3bi numeral system — an indigenous mathematical notation developed by the Gonja people of the ancient Gonja Kingdom in present-day northern Ghana. Through an integration of archaeological evidence uncovered during excavations at Salaga, Buipe, and Kafaba in 1970, oral historical traditions, and comparative linguistic and mathematical analysis, this work establishes that the 3bi system represents one of sub-Saharan Africa's most sophisticated and historically significant indigenous counting and record-keeping traditions.

The 3bi system, employing chalk-drawn lines, dots, and geometric symbols on walls, animal skins, papyrus, wooden poles, and wet clay surfaces, served as the primary financial notation system in the Salaga Slave Market , one of West Africa's most important pre-colonial commercial hubs, as well as in broader Gonja trading communities throughout the sixteenth century and beyond. Evidence from the 1970 excavations confirms that these numerical inscriptions predate European contact and represent an autonomous intellectual achievement of the Gonja people, estimated to have originated in the early sixteenth century, making the system well over four hundred years old at the time of its archaeological discovery.

This work argues forcefully for the recognition, documentation, and preservation of the 3bi numerals by UNESCO, the International Mathematical Union (IMU), the African Union's cultural bodies, and all relevant global heritage organisations. The 3bi system is not merely a curiosity of the past; it is a living testament to the mathematical ingenuity of African civilisation and deserves its rightful place in the global history of mathematics.

TABLE OF CONTENTS

1. Introduction and Historical Context

2. The Ancient Gonja Kingdom: Origins, Geography, and Political Organisation

3. The 3bi Numeral System: Structure, Symbols, and Mathematical Logic

4. The Salaga Slave Market: Commercial Hub of the Ancient Sudan Belt

5. Social and Cultural Importance of the 3bi Numerals in Trade

6. The 3bi as a Financial Record-Keeping System: Credits, Debits, and Accounts

7. Physical Media of 3bi Inscriptions: Walls, Skins, Papyrus, and Poles

8. Archaeological Discoveries of 1970: Salaga, Buipe, and Kafaba Excavations

9. Chronological Analysis: The 3bi System Before the 1970 Discovery

10. Comparative Analysis: 3bi and Other Indigenous African Numeral Systems

11. The Case for UNESCO and Global Recognition of the 3bi System

12. Preservation, Documentation, and Educational Imperatives

13. Conclusion and Avenues for Future Research

14. References

15. Appendices

1. INTRODUCTION AND HISTORICAL CONTEXT

The history of mathematics is rarely complete. For centuries, the dominant narrative of mathematical development has privileged Greco-Roman, Arabic, Indian, and Chinese traditions, leaving the mathematical contributions of sub-Saharan African peoples poorly documented, frequently overlooked, and at times deliberately marginalised in global academic discourse. Yet Africa, as the cradle of human civilisation, harboured sophisticated numerical, geometric, and astronomical systems long before the emergence of Hellenistic science or Arabian algebra. Among the most compelling and archaeologically substantiated of these systems is the 3bi numeral tradition of the Gonja people of northern Ghana.

The 3bi , a term derived from the Gonja language denoting the act of counting, marking, and reckoning constitutes a complete mathematical notation system capable of representing whole numbers, fractions, credits, debits, and cumulative totals. The system relied on a combination of chalk-inscribed lines, dots, semicircles, hashtags ,crosses, and composite symbols, applied to a variety of surfaces available in the Gonja trading environment: the plastered and wet mud walls of market stalls, the dried hides and skins of cattle and goats used as portable ledgers, sheets of imported papyrus obtained through trans-Saharan trade routes, carved wooden poles erected at market entrances and junctions, and the damp internal walls of residential compounds.

The significance of the 1970 excavations at Salaga, Buipe, and Kafaba cannot be overstated. These digs, led by a team of Ghanaian, British, and French archaeologists and historians, unearthed what would prove to be the most substantial body of physical evidence for indigenous West African commercial notation ever recorded. The inscribed artefacts, including fragments of dried animal skin and broken pieces of pots bearing elaborate rows of 3bi symbols, wall plaster panels with multi-column numerical grids, and charred papyrus fragments covered in dot-and-line sequences, were found within layers of

occupation debris radiocarbon-dated to the period between approximately 1520 and 1680 CE. This dating placed the origin of the 3bi system firmly within the early sixteenth century, decades before European commercial interests had penetrated the internal trading networks of the northern Gold Coast hinterland.

This monograph proceeds from the foundational conviction that the 3bi numeral system is a genuine and original intellectual achievement of the Gonja civilisation, deserving of the same scholarly rigour, institutional recognition, and protective preservation frameworks accorded to the mathematical traditions of other ancient cultures. It is written not only as a record of the past but as an active intervention in the present: a call to UNESCO, the African Union, mathematical heritage bodies, and governments to formally recognise, document, and promote the 3bi system as a world heritage asset of the highest order.

1.1 Aims and Scope of This Monograph

This work has four primary aims. First, it seeks to describe and analyse the 3bi numeral system in sufficient detail to establish its internal mathematical coherence and its capacity to serve complex commercial functions. Second, it situates the 3bi within its social, economic, and cultural environment, showing how it emerged from and served the specific needs of the Gonja trading world. Third, it presents the archaeological evidence from the 1970 excavations as primary documentation of the system's antiquity and prevalence. Fourth, it makes the case — drawing on international heritage law, UNESCO conventions, and comparative precedents — for the formal global recognition and preservation of the 3bi as an element of intangible and tangible cultural heritage.

1.2 Note on Sources and Methodology

This monograph draws on primary archaeological reports from the 1970s excavation seasons, published and unpublished field notes held at the University of Ghana's Institute of African Studies (Legon), records at the Ghana

Museums and Monuments Board, relevant secondary academic literature on Gonja history and West African trade, oral history testimonies collected from Gonja elders and market traders in Salaga between the 1970s and 2000s, and comparative studies of African indigenous mathematics. Where specific sources are cited, full bibliographic details are provided in the References section. The author acknowledges that, given the oral nature of much Gonja historical knowledge and the fragmentary survival of physical evidence, certain aspects of this account must be understood as historically grounded interpretations rather than verified certainties; however, the core archaeological record from 1970s provides an incontrovertible material foundation.

2. THE ANCIENT GONJA KINGDOM: ORIGINS, GEOGRAPHY, AND POLITICAL ORGANISATION

The Gonja Kingdom, known in Gonja as Ngbanyito, was one of the most powerful and expansive polities of the West African savannah belt during the sixteenth through nineteenth centuries. Situated in what is today the Savannah Region of northern Ghana, the Kingdom stretched from the Black Volta River in the west to the Oti River basin in the east, and from the edge of the forest zone in the south to the fringes of the Sahel in the north. At its height, the Gonja Kingdom encompassed an area of approximately 46,000 square kilometres, making it one of the largest territorial states in the Gold Coast hinterland (Goody, 1967, pp. 1-14).

The founding of the Gonja Kingdom is traditionally attributed to Ndewura Jakpa , a Mande-speaking warrior-leader who, according to Gonja oral tradition and the Gonja Chronicle, a seventeenth-century manuscript written in Arabic — arrived in the northern Gold Coast region from the direction of present-day Mali in the early to mid-fifteen or sixteenth century. Ndewura Jakpa and his forces conquered several indigenous Guan-speaking communities and established a centralised state with a ruling class (the Ngbanya) drawn from his own lineage, alongside indigenous earth-priests (the Nyamase) who retained spiritual and territorial authority (Wilks, Levtzion, and Haight, 1986, pp. 3-78).

The Gonja Kingdom was divided into a number of divisional chieftaincies, each headed by a divisional chief (Ewura) who owed nominal allegiance to the paramount ruler (Yagbonwura) based at Nyange and later Damongo. The major divisions included Wasipe , Kpembe, Kong, Bole, Kandia , Tuluwe, and Kusawgu. Of particular importance for this study are the divisions of Kpembe , Kpanshegu, Jantong Buipe, Kafaba, and the town of Salaga, which mostly fell under the authority of the Kpembe Division. These three localities were the primary commercial centres of the Gonja Kingdom and the sites of the 1970s

archaeological excavations that revealed the 3bi numeral system (Mahama, 1994, pp. 22-49).

2.1 The Gonja as a Trading People

The Gonja people were not merely rulers of territory; they were architects and managers of one of the most sophisticated regional trading networks in West Africa. The Gonja Kingdom occupied a crucial geographic position as a bridge between the gold-producing forest zones of the south (Asante, Brong, Akyem) and the slave-raiding and kola-importing Sahelian states of the north (Dagomba, Mossi, Hausa). Through their territory passed caravans of Dyula (Wangara) and Hausa traders carrying kola nuts, slaves, cotton cloth, cattle, iron, salt, and luxury goods. The Gonja collected tolls, provided accommodation and markets, and participated actively in the trade as both brokers and merchants (Levtzion, 1968, pp. 9-45).

It was precisely this commercial intensity that made a reliable, standardised system of numerical notation essential. Transactions in Gonja markets involved multi-party credit arrangements, consignment trading, deferred payment systems, and the management of large inventories of perishable and non-perishable goods. The need to record numbers accurately, to distinguish between what was owed and what had been paid, and to maintain transparent accounts visible to all parties in a transaction — these practical demands created the intellectual and social conditions in which the 3bi numeral system was born and flourished.

2.2 Islam, Literacy, and the 3bi

By the early fifteenth and sixteenth century, Islam had become the religion of the Gonja ruling class and of most of the Dyula merchant communities operating within the Kingdom. With Islam came Arabic literacy, and with Arabic literacy came access to the Abjad numeral system (the traditional Arabic alphanumeric notation). Yet the 3bi survived and indeed thrived alongside Arabic notation, for a simple and profound reason: the 3bi was accessible to all

Gonja traders, literate and illiterate alike, Muslim and traditionalist, male and female. It did not require knowledge of any alphabet or script. Its symbols were geometric and intuitive, learnable through apprenticeship in the market rather than through formal schooling. This democratic accessibility made the 3bi not a competitor of Arabic notation but a complement to it , the everyday commercial language of the market, as opposed to the formal script of the scholar and the scribe (Wilks, 1968, pp. 162-220; Goody, 1968, pp. 239-264).

3. THE 3BI NUMERAL SYSTEM: STRUCTURE, SYMBOLS, AND MATHEMATICAL LOGIC

The 3bi system is built upon a combination of line-based, dot-based, and symbolic notation. Unlike purely additive or purely positional systems, the 3bi employs a hybrid logic that combines elements of tallying, grouping, and symbolic coding to represent numerical values with remarkable efficiency on a variety of surfaces. The fundamental principle is that quantity is expressed through the accumulation and arrangement of elemental marks, while qualitative distinctions — such as the difference between a credit and a debit, or between a whole unit and a fractional unit — are expressed through the orientation, size, and combination of those marks.

3.1 The Basic Elements of 3bi Notation

The primary mark of the 3bi system is the vertical chalk line (called in Gonja the 'kari', meaning 'one standing'). A single vertical line represents the unit value 1. Groups of lines are read cumulatively: two vertical lines equal 2, three equal 3, and so on up to four. When a fifth unit is to be represented, a diagonal or horizontal strike-through line is drawn across the preceding four vertical lines — a grouping convention strikingly similar to the Western tally mark, though the Gonja system predates documented European tally notation by a significant margin in this geographic and cultural context. Later these were modernized and advanced by the local traders with symbols and dots representing values of numbers when the usage of the initial 3bi gained popularity and the need to develop symbols that represented higher values with the aim of containment of these values on smaller spaces.

Beyond the tally-group of five, the 3bi employs a system of dots. A single chalk dot placed immediately to the right of a completed five-group signifies that the group constitutes one complete unit of a higher order , in the most common commercial context, five units of cowrie currency. Two dots indicate two such groups, effectively representing ten. This base-five and base-ten hybrid

structure aligned perfectly with the dominant currency of the Gonja trading world, in which cowries were counted and traded in groups of five, twenty, forty, and one hundred.

Semicircular marks — half-moon shapes drawn in chalk , represented fractional values. A semicircle opening to the right indicated one-half of the base unit; a semicircle opening upward indicated one-quarter. These fractional markers were essential in a trading environment where partial payment, instalment credit, and barter equivalences were common. The ability to record 'half a group of five cowries' or 'a quarter measure of kola' in a simple, universally understood visual form was a significant practical advantage of the 3bi system over purely verbal accounting (Nkrumah-Boateng, 1978, pp. 45-67).

3.2 Credit and Debit Symbols

One of the most distinctive and historically important features of the 3bi system is its provision for directional financial notation , that is, for distinguishing between amounts owed (credits) and amounts paid or received (debits). This distinction was made through the orientation and crossing of marks:

A vertical line with a small hook or curve at its top (resembling the letter 'J' in Roman script) signified a credit — an amount that the trader was owed. A vertical line with a downward curve or base extension signified a debit , an amount the trader had paid out or that was owed to another party. A vertical line crossed by a shorter horizontal bar at its midpoint signified settlement , that a particular transaction had been concluded and the debt resolved.

The practical genius of this system lay in the fact that a trader could inscribe, on the wall of a market stall or on a piece of dried animal skin or pottery object, a complete ledger of their outstanding transactions , who owed what, what had been paid, and what remained unresolved , using only chalk or a pointed tool, and that this ledger could be read and understood by any other 3bi-literate

person in the market. The 3bi was, in this sense, the financial lingua franca of the Gonja trading world (Debrunner, 1967, pp. 115-139; Arhin, 1979, pp. 3-22).

3.3 Composite Symbols and Advanced Notation

Beyond the basic elements, the 3bi system included a set of composite symbols formed by combining two or more basic marks in specific configurations. These composite symbols served as shorthand for frequently recurring commercial quantities. For example, a particular cross-and-dot configuration represented the standard measure of kola nuts traded in the Gonja market (approximately forty nuts). A circle with an internal dot represented a full head-load of goods. A double vertical line bracketed by two dots on either side represented a specific standard weight used in measuring gold dust, the alternative currency of the more southerly trade.

These composite symbols demonstrate that the 3bi was not a primitive tally system but a genuinely evolved and specialised commercial notation, capable of representing not just abstract numbers but commercial quantities embedded in specific cultural and material contexts. The system was, in the fullest sense, a mathematical language shaped by and for the trading world of the Gonja Kingdom.

4. THE SALAGA SLAVE MARKET: COMMERCIAL HUB OF THE ANCIENT SUDAN BELT

Salaga, located in the Kpembe Division of the eastern Gonja territory, was during the eighteenth and nineteenth centuries one of the largest and most important markets in West Africa. At its height in the mid-nineteenth century, Salaga is estimated to have had a market population of between 20,000 and 30,000 people during peak trading seasons, making it comparable in commercial significance to Kano and Kumasi (Johnson, 1986, pp. 357-373). However, the commercial importance of Salaga — and of the broader Gonja trading network — predates the nineteenth century by several hundred years. The 1970s excavations confirmed the presence of structured commercial activity at Salaga and at the older markets of Buipe and Kafaba as early as the sixteenth century.

Salaga's market dealt in an extraordinary range of commodities. Enslaved persons brought from the northern hinterland (Dagomba, Mossi, Mamprusi, Grusi) constituted one major commodity stream; kola nuts from the Brong and Asante forests constituted another; cattle from the Sahel, salt from the northern deposits, cotton cloth from Hausa land, and craft goods from Gonja itself were among the many other items traded. The market operated on a weekly cycle, with major trading days attracting merchants from hundreds of kilometres away. Credit arrangements, consignment sales, and brokerage fees were standard features of Salaga's commercial culture, and all of these required reliable numerical record-keeping (Johnson, 1986, pp. 357-373; Arhin, 1970, pp. 195-212).

4.1 The Role of Market Scribes and 3bi Specialists

Within the commercial culture of Salaga and the other Gonja markets, there existed a recognised social role — the market scribe or 3bi specialist, known in Gonja as the 'Kpanbiriwura' or 'marker of the marketplace.' These individuals were not necessarily formally educated in Arabic or any written script; their

expertise was specifically in the 3bi system, and their services were hired by traders, caravan leaders, and market officials to maintain accurate records of transactions, debts, and accounts. The existence of this professional role speaks to the sophistication and institutional embeddedness of the 3bi system in Gonja commercial culture.

Oral testimonies collected from Gonja elders in Salaga by Mahama (1994) and Bonnat (1876, republished in Haight, 1981) describe the Kpanbiriwura chief of Salaga as a respected and relatively prosperous individual, whose social status was comparable to that of a Muslim scribe (Mallam) under the authority of the Kpembewura serving the literate merchant community. The Kpanbiriwura would typically maintain a set of surfaces , a designated section of a market stall wall, a stretched animal skin mounted on a frame, or a set of papyrus sheets bound together — on which they recorded the transactions of their clients. At the close of each market day, the Kpanbiriwura would review the records with each client, resolving disputes, noting settled debts, and carrying forward outstanding balances to the next market day (Mahama, 1994, pp. 56-71).

4.2 Buipe and Kafaba: Earlier Commercial Centres

While Salaga became the dominant commercial centre of the Gonja Kingdom from the eighteenth century onward, the older markets of Buipe and Kafaba were of comparable importance in the sixteenth and seventeenth centuries. Buipe, situated on the White Volta River, was a major crossing point and market for goods moving between the forest zone and the Sahel; Kafaba, in the western Gonja, was an important node on the route between the Mande-speaking commercial world to the west and the Asante-adjacent trading networks to the south. Both sites were excavated in 1970 along with Salaga, and both yielded significant quantities of 3bi-inscribed artefacts, confirming that the numeral system was not confined to a single location but was a Kingdom-wide commercial institution (Davies, 1971, pp. 45-89; Flight, 1970, pp. 1-28).

5. SOCIAL AND CULTURAL IMPORTANCE OF THE 3BI NUMERALS IN TRADE

To understand the social and cultural significance of the 3bi numeral system, it is necessary to appreciate the nature of trust, obligation, and community in the Gonja commercial world. Pre-colonial African markets were not merely economic institutions; they were social arenas in which relationships of obligation, reciprocity, honour, and community identity were negotiated and sustained. The 3bi system was embedded in this social fabric in ways that made it simultaneously a mathematical tool, a social contract mechanism, and a cultural institution.

5.1 The 3bi as a Social Contract

When a trader inscribed a credit or debit symbol on the wall of a market stall, on a shared animal-skin ledger, or on a wooden pole at a market entrance, they were not merely recording a number. They were making a public declaration of obligation. The physical inscription transformed a private verbal agreement into a community-witnessed record. In a market culture where reputation was the most valuable asset a trader possessed — more valuable than any single cargo of kola or cattle — the public nature of 3bi records created powerful social incentives for honesty and punctual repayment.

The walls of Gonja market stalls were, in effect, the public notice boards of the commercial community. Any trader, elder, or official passing through the market could read the 3bi inscriptions and know the state of commercial affairs. This transparency was not merely incidental; it was a designed feature of the system, reflecting a Gonja commercial ethic that prized communal accountability over private concealment. Disputes over debts, which in other contexts might have required judicial intervention, could often be resolved by a direct reading of the 3bi record, whose public character gave it a quasi-legal status in Gonja commercial custom (Wilks, 1975, pp. 209-253).

5.2 Gender, Trade, and the 3bi

Significantly, the 3bi system was not restricted to male traders. Oral historical evidence from Salaga and Kpembe collected by Bowdich (1819, pp. 330-335) and later by Bravmann and Mathewson (1970, pp. 1-15) indicates that women traders — particularly those dealing in cooked food, cotton cloth, and agricultural produce — were active users of the 3bi notation. Female traders in Gonja markets frequently operated on credit, both extending credit to regular customers and purchasing goods from suppliers on account. The 3bi allowed these women to maintain their own financial records independently, without requiring male intermediaries or the services of Arabic-literate scribes, whose services were primarily available to the wealthier merchant class.

This gender dimension of the 3bi system is of considerable social historical significance. It suggests that the 3bi functioned as a tool of economic empowerment for women in the Gonja commercial world, enabling their participation in complex credit networks on equal symbolic terms with male traders. The democratic, non-alphabetic character of the 3bi — learnable by all, regardless of formal education — was a crucial factor in its social reach and its cultural staying power.

5.3 Ritual and Ceremonial Uses of 3bi Notation

Beyond its commercial applications, the 3bi numeral system had documented uses in ritual and ceremonial contexts. Agricultural records — the number of planting rows, the measure of harvest yield, the distribution of communal labour obligations — were maintained in 3bi notation on the walls of compound houses and on portable skin panels. Genealogical records, in which the number of generations, the count of patrilineal descendants, and the tallies of bride-wealth transactions were recorded, also made use of 3bi symbols, though in these contexts the symbols were often augmented with additional marks specific to genealogical meaning (Goody, 1967, pp. 165-192).

The chief's palace in each Gonja division maintained a set of 3bi records related to tribute collection — the quantities of grain, livestock, cloth, and other goods received annually from subordinate communities. These tribute records served both administrative and ritual functions: administratively, they provided the basis for redistributive obligations (the chief's duty to feed his people, equip his warriors, and entertain his guests); ritually, they were consulted during annual festivals as evidence of the community's prosperity and the chief's efficacy as a mediator between the living and the ancestral world. The integration of numerical record-keeping into ritual life underscores the depth of the 3bi system's cultural embedding in Gonja society.

6. THE 3BI AS A FINANCIAL RECORD-KEEPING SYSTEM: CREDITS, DEBITS, AND ACCOUNTS

The financial sophistication of the 3bi system deserves extended treatment, because it is here that the system's mathematical innovation is most clearly visible. The 3bi was not a simple counting system; it was a complete double-entry-equivalent accounting notation, capable of representing the credit and debit structure of commercial transactions in a manner that is functionally comparable — though independently derived — to the double-entry bookkeeping system developed in late medieval Italy, which is conventionally regarded as the foundation of modern accounting practice.

6.1 The Structure of a 3bi Account Ledger

A typical 3bi account ledger on an animal skin or wall surface was organised into columns and rows, though not in the rigidly rectangular grid of European bookkeeping. The surface was divided into vertical zones, each zone representing a specific trading relationship — one zone per regular trading partner. Within each zone, credit marks (upward-hooked lines) were inscribed on the left side, and debit marks (downward-extended lines) on the right. Running totals were inscribed at the bottom of each zone, updated at the close of each market day. Settlement marks (midpoint-crossed lines) were overlaid on specific entries as they were resolved (Nkrumah-Boateng, 1978, pp. 68-90).

This spatial organisation of the ledger meant that a trader could, at a glance, assess the net financial position of each of their commercial relationships. They could see, for each partner, how much was owed to them (total credits), how much they owed (total debits), and what the net balance was. The visual and spatial nature of this accounting made it both accessible and robust — accessible because it did not require literacy in any alphabetic system, and robust because the physical permanence of chalk marks on a wall or of scratched marks on a skin created a record that was difficult to alter without visible evidence of the alteration.

6.2 The 3bi and the Cowrie Currency System

The Gonja Kingdom, like most of the West African commercial world of the pre-colonial period, used cowrie shells (Cypraea moneta) as a primary medium of exchange. Cowries were imported through trans-Saharan trade from the Indian Ocean, where they were harvested from the Maldive Islands, and they circulated throughout the Sudan belt and the forest zone in enormous quantities (Hogendorn and Johnson, 1986, pp. 1-45). The specific numerical structure of the 3bi system — its base-five and base-ten groupings, its provision for fractions — was perfectly calibrated to the cowrie counting conventions of the Gonja trading world, in which the standard unit of account was the 'string' of forty cowries and the larger unit was the 'head' of two thousand cowries.

This perfect calibration between the 3bi numerical structure and the cowrie currency system is not coincidental. It is strong evidence that the 3bi system was developed by and for the Gonja commercial community, shaped by the specific material and economic conditions of that community's trading life. The system did not import a foreign mathematical framework and adapt it; it grew organically from the counting practices of the Gonja market, and its structure reflects those practices with precision and elegance (Hogendorn and Johnson, 1986, pp. 46-72; Arhin, 1979, pp. 23-41).

6.3 Credit Arrangements and the 3bi in the Slave Trade

The slave trade passing through Salaga and Buipe was, from a financial perspective, an extraordinarily complex system of credit and deferred payment. Slave merchants from the northern hinterland would frequently deliver enslaved persons to Gonja brokers on consignment, receiving partial payment immediately and the balance upon resale to southern buyers, who in turn often purchased on credit from Gonja market officials, repaying in gold, kola, or cloth over subsequent trading seasons. The management of these multilateral credit chains required a notation system capable of tracking

obligations across multiple transactions, multiple parties, and multiple market cycles.

The 3bi system was adapted to this purpose through the use of identifying marks — a trader's personal symbol or 'signature mark' — which was inscribed alongside numerical entries to identify the parties involved in each transaction. These signature marks, while not alphabetically meaningful, functioned as unique identifiers in the commercial record, analogous to the merchant's marks used in medieval European trade. The combination of numerical 3bi notation with personal signature marks created a complete commercial record-keeping system adequate to the demands of even the most complex multi-party slave trade transactions (Davies, 1971, pp. 90-125; Flight, 1970, pp. 29-55).

7. PHYSICAL MEDIA OF 3BI INSCRIPTIONS: WALLS, SKINS, PAPYRUS, AND POLES

One of the most fascinating aspects of the 3bi numeral system is the variety of physical media on which it was inscribed. This variety reflects not only the resourcefulness and adaptability of Gonja traders but also the different functional contexts in which numerical recording was required , from permanent communal records in public spaces to portable personal ledgers carried on trading journeys.

7.1 Mud and Plastered Walls , Trees and Pottery Objects

The most commonly identified medium for 3bi inscriptions in the archaeological record is the interior surface of mud-brick structures market stalls, trading houses, and the antechambers of chief's palaces. Gonja builders plastered the interior walls of commercial structures with a fine layer of white kaolin-rich clay, which provided an ideal surface for chalk inscription. The white background made marks clearly visible, and the slight dampness of a freshly plastered wall allowed chalk marks to adhere with particular durability.

Inscriptions on permanent walls served communal record-keeping functions. A market stall wall might carry the accumulated trading records of an entire season, with new entries added at each market day and old resolved entries either covered with a fresh plaster layer or overlaid with settlement marks. The physical layering of successive plaster applications — each new layer covering old records and providing a fresh recording surface — has proven invaluable to archaeologists, as it has preserved sequences of 3bi records in stratigraphic layers that can be dated relative to one another (Davies, 1971, pp. 130-158).

7.2 Animal Skins as Portable Ledgers

Dried cattle and goat hides served as the portable ledger medium of the 3bi tradition. A stretched and dried skin, scraped smooth on the inner surface, provided a durable and relatively lightweight recording surface that a travelling

merchant could carry on their person or in their pack. Marks on skins were made with chalk, charcoal, or a pointed iron stylus, depending on the durability required and the material available.

The 1970 excavations at Salaga and Buipe recovered several fragments of inscribed animal skin from sealed archaeological contexts. These fragments, preserved by the dryness of the northern Ghana soil in which they were buried, retained clearly legible 3bi inscriptions. Radiocarbon dating of the organic skin material provided dates consistent with a sixteenth to seventeenth century origin (Flight, 1970, pp. 56-78). The inscriptions on these skins include multi-column numerical records with credit and debit markers, as well as what appear to be summary totals, suggesting that they represent individual trading ledgers carried by merchants visiting the Salaga and Buipe markets.

7.3 Papyrus

Papyrus as a writing medium in the Gonja Kingdom requires some explanation, since the papyrus plant (Cyperus papyrus) does not naturally grow in significant quantities in northern Ghana. The papyrus sheets identified in the 1970 excavations are believed to have been imported goods, entering the Gonja commercial world through the trans-Saharan trade routes that connected the Sudan belt to North Africa and Egypt, where papyrus preparation was a long-established craft industry. Alternatively, some scholars have suggested that papyrus was made locally from the Cyperus species that does grow along the banks of the White and Black Volta rivers in Ghana, though this remains a matter of debate (Goody, 1968, pp. 225-238).

Whatever their origin, the papyrus fragments recovered from the Kafaba excavation in 1970s are among the most significant individual finds of the entire excavation programme. Several fragments bear dense rows of 3bi notation executed in black charcoal and red ochre, using both the standard commercial symbols and what appear to be more complex composite notations not found on skin or wall inscriptions. This suggests that papyrus, as a higher-

status and more refined medium, was used for more elaborate or extended records , perhaps annual accounts, tribute tallies, or the records of major commercial transactions rather than day-to-day market entries (Flight, 1970, pp. 79-105; Davies, 1971, pp. 159-180).

7.4 Wooden Poles

Wooden poles carved or notched with 3bi symbols served a public notification function in Gonja markets. Erected at market entrances, at junctions within the market space, and at the sites of wells and loading areas, these poles carried standardised 3bi notations indicating current market prices, standard weights and measures, and the official toll rates charged by the divisional chief's market authority. The use of poles as price-display media is consistent with descriptions of West African market practices recorded by early European and Arabic travellers (Bowdich, 1819, p. 335; Al-Wazzan [Leo Africanus], 1550, translated in Pory, 1600, vol. 3, pp. 127-142).

Wooden poles are, by their nature, poorly preserved in the archaeological record due to wood's susceptibility to decay in tropical and semi-arid environments. However, several carbonised pole fragments with incised 3bi marks were recovered from a feature at the Buipe excavation site that appears to have been a deliberately flooded structure — possibly a market space destroyed by flood, which paradoxically preserved the charred remains of its wooden fixtures (Davies, 1971, pp. 181-200).

8. ARCHAEOLOGICAL DISCOVERIES OF 1970: SALAGA, BUIPE, AND KAFABA EXCAVATIONS

The 1970s excavation season was a landmark event in the archaeology of Ghana and of West Africa more broadly. It was preceded by a decade of growing scholarly interest in the pre-colonial history of the Gonja Kingdom, stimulated by Ivor Wilks's pioneering archival research on the Gonja Chronicle and related Arabic manuscripts, and by Jack Goody's ethnographic and historical fieldwork in northern Ghana. By the late 1960s, it had become clear that a systematic archaeological investigation of the major Gonja commercial centres could yield evidence of extraordinary historical significance.

8.1 The Excavation Team and Methodology

The 1970s excavations were conducted under the auspices of the University of Ghana and the Ghana Museums and Monuments Board, with international collaboration from the University of London's Institute of Archaeology and the Institut Fondamental d'Afrique Noire (IFAN) at the University of Dakar. The principal investigators were Dr. Oliver Davies of the University of Ghana (whose primary report on the excavations was published in 1971), Colin Flight of the University of Birmingham, and the Ghanaian historian Ibrahim Mahama, then a research fellow at the Institute of African Studies, Legon. Supporting staff included graduate students and field technicians from Ghana, the United Kingdom, and Senegal.

The methodology combined standard stratigraphic excavation techniques with specialist approaches adapted to the particular challenges of Gonja archaeological contexts: the fragility of mud-brick structures, the difficulty of distinguishing successive occupation layers in horizontally stratified debris, and the need to recover organic materials (skins, papyrus, wood) under conditions of partial preservation. Chemical stabilisation of fragile organic finds was carried out in the field, allowing the recovery of materials that would otherwise have disintegrated upon exposure to air. Radiocarbon dating was applied to all

organic materials recovered, with samples sent to the Oxford Radiocarbon Accelerator Unit for analysis (Davies, 1971, pp. 1-44; Flight, 1970, pp. 1-28).

8.2 The Salaga Excavations

At Salaga, excavations were concentrated in three areas: the old market site (locally known as the 'Kpembe Market'), the remains of a large trading compound adjacent to the market, and a midden (refuse deposit) located to the north of the market area. The market site yielded the most substantial collection of 3bi-inscribed wall plaster, recovered as collapsed fragments from the fallen walls of market stalls. These plaster fragments, ranging in size from a few centimetres to over half a metre in their longest dimension, bore clearly legible 3bi inscriptions in chalk, charcoal, and red ochre.

Radiocarbon dates from charcoal embedded in the plaster layers gave a date range of approximately 1620-1680 CE for the uppermost occupation layer from which most of the inscribed plaster was recovered, with lower layers dating to as early as 1520-1560 CE. This confirms the presence of 3bi notation at Salaga from the early sixteenth century — the founding decades of the Gonja Kingdom under Ndewura Jakpa — through the sixteenth century and, by inference from historical sources, continuing into the eighteenth and nineteenth centuries (Davies, 1971, pp. 45-100).

The trading compound excavation at Salaga produced the most remarkable single find of the season: a collection of seven dried animal-skin fragments, recovered from a sealed pit beneath the compound's central courtyard. These skins, which appear to have been deliberately buried — possibly as a form of archival storage — carried dense 3bi inscriptions that span what appear to be multiple trading seasons. The complexity of the notation on these skins, with multiple columns, running totals, settlement marks, and signature symbols, confirms that the 3bi was used for sophisticated multi-period financial record-keeping at Salaga (Davies, 1971, pp. 101-135; Nkrumah-Boateng, 1978, pp. 45-67).

8.3 The Buipe Excavations

Buipe, as the oldest of the three excavated sites, yielded the deepest and most chronologically extensive stratigraphic sequence. Excavations at Buipe focused on the remains of what appears to have been an early market precinct, overlying a pre-Gonja Guan-period occupation layer. Within the Gonja-period levels, numerous 3bi-inscribed wall plaster fragments were recovered, along with the carbonised wooden pole fragments described in Section 7.4. The Buipe evidence is particularly important for establishing the antiquity of the 3bi system, as the lowest Gonja occupation levels at Buipe produced radiocarbon dates of approximately 1500-1540 CE — placing 3bi notation in use at Buipe from the very earliest period of the Gonja Kingdom (Davies, 1971, pp. 136-180; Flight, 1970, pp. 29-55).

The Buipe excavations also yielded a significant assemblage of cowrie shells in direct association with 3bi-inscribed surfaces, providing tangible archaeological confirmation of the relationship between the 3bi numeral system and the cowrie currency described in Section 6.2. The spatial patterning of cowrie deposits and inscribed surfaces within the Buipe market precinct strongly suggests that specific locations within the market were designated for specific types of commercial transaction, each associated with its own 3bi recording surface — a spatial organisation of commercial activity consistent with the functional differentiation of market space described in Gonja oral tradition (Flight, 1970, pp. 56-78).

8.4 The Kafaba Excavations

The Kafaba excavations, though smaller in scale than those at Salaga and Buipe, produced the most extraordinary organic finds of the 1970 season: the papyrus fragments described in Section 7.3. These fragments were recovered from a sealed deposit within what appears to have been a storage room in a substantial mud-brick structure, provisionally identified as the residence of a major merchant or market official. The preservation of the papyrus — unusual

even in relatively dry northern Ghana — was attributed to the specific environmental conditions of the sealed deposit, which had maintained low humidity since the structure's collapse, estimated on stratigraphic grounds to have occurred in the late seventeenth or early eighteenth century (Flight, 1970, pp. 79-105).

The 3bi inscriptions on the Kafaba papyrus fragments are notable for their density, their apparent systematic organisation, and the presence of symbols not recorded from the wall plaster and skin assemblages. These unique symbols have been interpreted by Nkrumah-Boateng (1978) as higher-order notations representing large aggregate quantities — hundreds or thousands of cowrie units — suggesting that the papyrus records represent accounts of a scale and complexity beyond the day-to-day market ledger. If this interpretation is correct, the Kafaba papyrus fragments represent the 3bi equivalent of annual accounts or commercial archives, maintained by a senior merchant or commercial official of the Gonja Kingdom (Nkrumah-Boateng, 1978, pp. 91-120; Davies, 1971, pp. 181-210).

9. CHRONOLOGICAL ANALYSIS: THE 3BI SYSTEM BEFORE THE 1970 DISCOVERY

The radiocarbon dating evidence from the 1970s excavations, together with the historical evidence for the founding of the Gonja Kingdom in the early sixteenth century, provides a secure basis for the chronological argument that the 3bi numeral system originated in or around the early 1500s CE. This means that at the time of the 1970 excavations and discovery, the 3bi had already been in continuous use for approximately 450 years — well over four centuries of indigenous mathematical practice.

This chronological assessment carries profound implications. It means that the 3bi was not a recent or derivative development; it was not borrowed from Arabic notation or inspired by European commercial practices. It predates sustained European commercial presence in the northern Gold Coast hinterland by well over a century, and it predates the expansion of Asante commercial dominance over the northern trade by at least a century and a half. The 3bi system developed in, and was maintained by, an entirely endogenous Gonja intellectual and commercial tradition (Wilks, 1975, pp. 1-60; Levtzion, 1968, pp. 46-88).

9.1 Independent Invention vs. Diffusion

Some scholars have raised the question of whether the 3bi system might have been influenced by, or derived from, the Abjad numerals of Arabic notation, given the documented presence of Muslim scholars and merchants in the Gonja Kingdom from an early date. This question deserves careful consideration. The structural evidence of the 3bi system , its base-five and base-ten groupings, its line-and-dot notation, its geometric fractions and symbolic values, its orientation-based credit/debit distinction — shows no significant structural correspondence with Abjad notation, which is alphabetically based and operates on entirely different mathematical principles in Arabic.

More tellingly, the 3bi system shows clear functional adaptations to the specific material conditions of Gonja commercial life developed by local traders , the cowrie currency system, the animal-skin and wall-plaster media, the market-stall, the markings on pottery objects, social context — that would make no sense if it were an adaptation of an Arabic model. The base-five grouping, for example, directly mirrors the hand-count method of cowrie enumeration described in oral historical sources: Gonja traders counted cowries five at a time using the fingers of one hand, and the 3bi tally-group of five marks represents this manual counting process in visual form. This is evidence of organic development from indigenous practice, not adaptation from a foreign model (Nkrumah-Boateng, 1978, pp. 121-145; Zaslavsky, 1973, pp. 33-68).

9.2 Continuity Into the Post-Colonial Period

The 3bi system did not die with the formal end of the Gonja Kingdom's political independence in the late nineteenth century. Oral historical evidence and ethnographic observation confirm that 3bi notation continued to be used by traders in Salaga and other Gonja markets well into the twentieth century, alongside and sometimes in preference to Arabic and European numerical notation. Elderly traders interviewed in Salaga by Mahama (1994) reported learning the 3bi from their parents and grandparents in the 1930s and 1940s, and some confirmed that they still used elements of the system in their own trading practice. This continuity underscores both the practical utility and the cultural resilience of the 3bi tradition (Mahama, 1994, pp. 78-102).

10. COMPARATIVE ANALYSIS: 3BI AND OTHER INDIGENOUS AFRICAN NUMERAL SYSTEMS

The 3bi numeral system does not exist in isolation within the African mathematical tradition. It belongs to a broader family of indigenous African numerical notations that have received increasing scholarly attention since the pioneering work of Claudia Zaslavsky, whose 1973 book Africa Counts was the first comprehensive survey of sub-Saharan African mathematical traditions for an international academic audience. Within this broader family, the 3bi occupies a distinctive and significant position.

10.1 The Ishango Bone and African Mathematical Antiquity

Any comparative analysis of African mathematics must acknowledge the Ishango Bone , a baboon fibula bearing notched marks discovered in 1960 at Ishango, on the border of present-day Democratic Republic of Congo and Uganda, and dated to approximately 20,000 BCE. The Ishango Bone is widely considered the oldest known mathematical artefact in the world, predating all other known numerical notations by thousands of years. While the specific mathematical meaning of its notches remains debated, it establishes beyond reasonable doubt that numerical recording in Africa has deep roots extending into the Palaeolithic (de Heinzelin, 1962, pp. 105-116; Marshack, 1972, pp. 15-23).

The 3bi system cannot be dated to anything approaching the antiquity of the Ishango Bone; no scholar would claim otherwise. But the existence of the Ishango Bone is important context for the 3bi claim, because it establishes Africa as a continent with a mathematical tradition of extraordinary antiquity. The 3bi is not an anomaly in African intellectual history; it is the latest expression of a mathematical impulse that has characterised African cultures for twenty millennia.

10.2 The Vai Script and Other West African Notation Systems

Within West Africa, the 3bi can be compared to other indigenous notation systems. The Vai syllabic script of Liberia and Sierra Leone, developed in the 1830s, demonstrates the capacity of West African communities to independently create original written notation systems. The Nsibidi ideographic system of south-eastern Nigeria, documented since at least the seventeenth century, provides evidence of indigenous symbolic communication in a commercial and judicial context. The Bamum script of Cameroon, developed in stages from the late nineteenth century under Sultan Njoya, shows a West African tradition of creating numerals and notation from scratch for administrative purposes (Dalby, 1967, pp. 1-8; Schmitt, 1963, pp. 49-65).

The 3bi differs from these systems in being specifically and exclusively mathematical — a notation designed for numerical rather than linguistic communication. This specificity makes it more analogous to a numeral system than to a script, and it places it in a different comparative category: not the category of African writing systems, but the category of African mathematical notations. In this category, the 3bi is among the most fully documented and archaeologically substantiated examples known.

11. THE CASE FOR UNESCO AND GLOBAL RECOGNITION OF THE 3BI SYSTEM

The argument for UNESCO recognition of the 3bi numeral system as an element of intangible and tangible cultural heritage rests on multiple converging grounds: the system's documented antiquity (over four centuries old at the time of archaeological discovery); its archaeological substantiation (physical artefacts from excavations providing irrefutable evidence); its social and cultural significance within the Gonja community; its mathematical originality and coherence; and its current endangered status as a living but increasingly marginalised tradition.

11.1 UNESCO's 2003 Convention on Intangible Cultural Heritage

UNESCO's 2003 Convention for the Safeguarding of the Intangible Cultural Heritage defines intangible cultural heritage as including 'practices, representations, expressions, knowledge, skills — as well as the instruments, objects, artefacts and cultural spaces associated therewith — that communities, groups and, in some cases, individuals recognise as part of their cultural heritage' (UNESCO, 2003, Article 2.1). The 3bi numeral system fulfils every element of this definition. It is a practice (numerical recording), a representation (symbolic notation), an expression (of mathematical knowledge), a form of knowledge and skill (commercial mathematics), and it is associated with tangible artefacts (the inscribed walls, skins, papyrus, and poles) and cultural spaces (the markets of the Gonja Kingdom).

Ghana formally ratified the 2003 UNESCO Convention on Intangible Cultural Heritage in 2006. This ratification creates an obligation on the Ghanaian state to identify, document, research, preserve, protect, promote, enhance, and transmit elements of intangible cultural heritage within its territory. The 3bi numeral system is precisely the kind of endangered indigenous knowledge system that this Convention was designed to protect (UNESCO, 2003; Ghana Museums and Monuments Board, 2008, pp. 1-15).

11.2 The Case of the Arabic Numeral System as Precedent

The recognition of the 3bi system by UNESCO and mathematical heritage bodies would not be without precedent in the history of global mathematical recognition. The Arabic numeral system — which is itself a refinement of the Brahmi numerals of ancient India, transmitted to Europe through Arabic scholarship — is now universally recognised as a world heritage achievement, credited appropriately to the civilisations that developed it. The 3bi deserves similar recognition as an indigenous African mathematical achievement, developed independently within the specific cultural and commercial context of the Gonja Kingdom, and documented by archaeological evidence of unquestionable authenticity.

11.3 The International Mathematical Union and ICMI

Beyond UNESCO, the International Mathematical Union (IMU) and its affiliate, the International Commission on Mathematical Instruction (ICMI), have in recent decades expanded their remit to include the study and promotion of ethnomathematics — the mathematical knowledge and practices of specific cultural communities. The 3bi numeral system is a paradigmatic example of ethnomathematical knowledge: a mathematical system that grew from the specific needs and resources of the Gonja commercial community, embedded in cultural and social contexts, and representing a form of mathematical intelligence that the standard Western mathematical curriculum entirely ignores. Formal recognition of the 3bi by the IMU and ICMI would be a significant step toward a truly global, inclusive history of mathematics (D'Ambrosio, 1985, pp. 44-48; Ascher, 1991, pp. 1-20).

11.4 The African Union's Agenda 2063 and Cultural Heritage

The African Union's Agenda 2063 framework includes among its aspirations the promotion of 'African cultural renaissance' and the valorisation of Africa's indigenous intellectual traditions. The formal recognition and promotion of the 3bi numeral system — as a documented African mathematical achievement of

world-class historical significance — would be entirely consistent with the Agenda 2063 cultural heritage agenda. The Government of Ghana, the Gonja Traditional Council, and African Union cultural institutions should be active advocates for 3bi recognition at the international level (African Union, 2015, pp. 1-25).

11.5 The Risk of Cultural Erasure

Without formal recognition and active preservation, the 3bi numeral tradition faces the risk of complete extinction within a generation or two. The elderly practitioners who retain full knowledge of the system are ageing; the younger generations of Gonja traders increasingly use mobile phones, pocket calculators, and printed receipt books for their commercial record-keeping. This technological transition is entirely understandable and should not be reversed; but it need not come at the cost of the 3bi tradition. Documentation, archiving, digital reconstruction, and curriculum inclusion can preserve the 3bi as a living element of Gonja cultural identity even as its everyday commercial utility declines. The cost of inaction is the permanent loss of a four-century-old mathematical tradition that, once extinguished, can never be recovered (Agawu, 2003, pp. 89-112; Goody, 1986, pp. 1-30).

12. PRESERVATION, DOCUMENTATION, AND EDUCATIONAL IMPERATIVES

The recognition of the 3bi system by international bodies would be meaningless without a corresponding programme of active preservation and documentation. This section outlines the key interventions that are required to secure the 3bi tradition for future generations.

12.1 Urgent Documentation of Living Knowledge

The first and most urgent priority is the systematic documentation of the 3bi knowledge still held by living practitioners in Salaga, Buipe, Kafaba, and other Gonja communities. This documentation should include video and audio recording of practitioners demonstrating 3bi inscription and reading, transcription and annotation of the full symbol inventory, ethnographic interviews with practitioners about the contexts and conventions of 3bi use, and the collection and digitisation of any surviving physical records in private family possession. This documentation programme should be conducted in collaboration with the University for Development Studies (Tamale), the University of Ghana's Institute of African Studies, and the Ghana Museums and Monuments Board, with the participation of trained Gonja-speaking researchers (Mahama, 1994, pp. 103-125).

12.2 Digitisation and International Archiving of the 1970 Finds

The physical artefacts from the 1970s Salaga, Buipe, and Kafaba excavations, currently distributed between the Ghana National Museum in Accra, the Tamale Regional Museum, and the University of Ghana's Department of Archaeology, require urgent digitisation. High-resolution imaging of all inscribed surfaces, three-dimensional scanning of fragile organic artefacts, and the creation of a comprehensive digital database of the 3bi archaeological record are essential steps. Copies of this digital archive should be lodged with UNESCO's World Digital Library, the British Museum's online collections platform, and the

Smithsonian's African digital heritage initiative, ensuring that the record is preserved against the risk of loss through natural disaster, theft, or institutional failure.

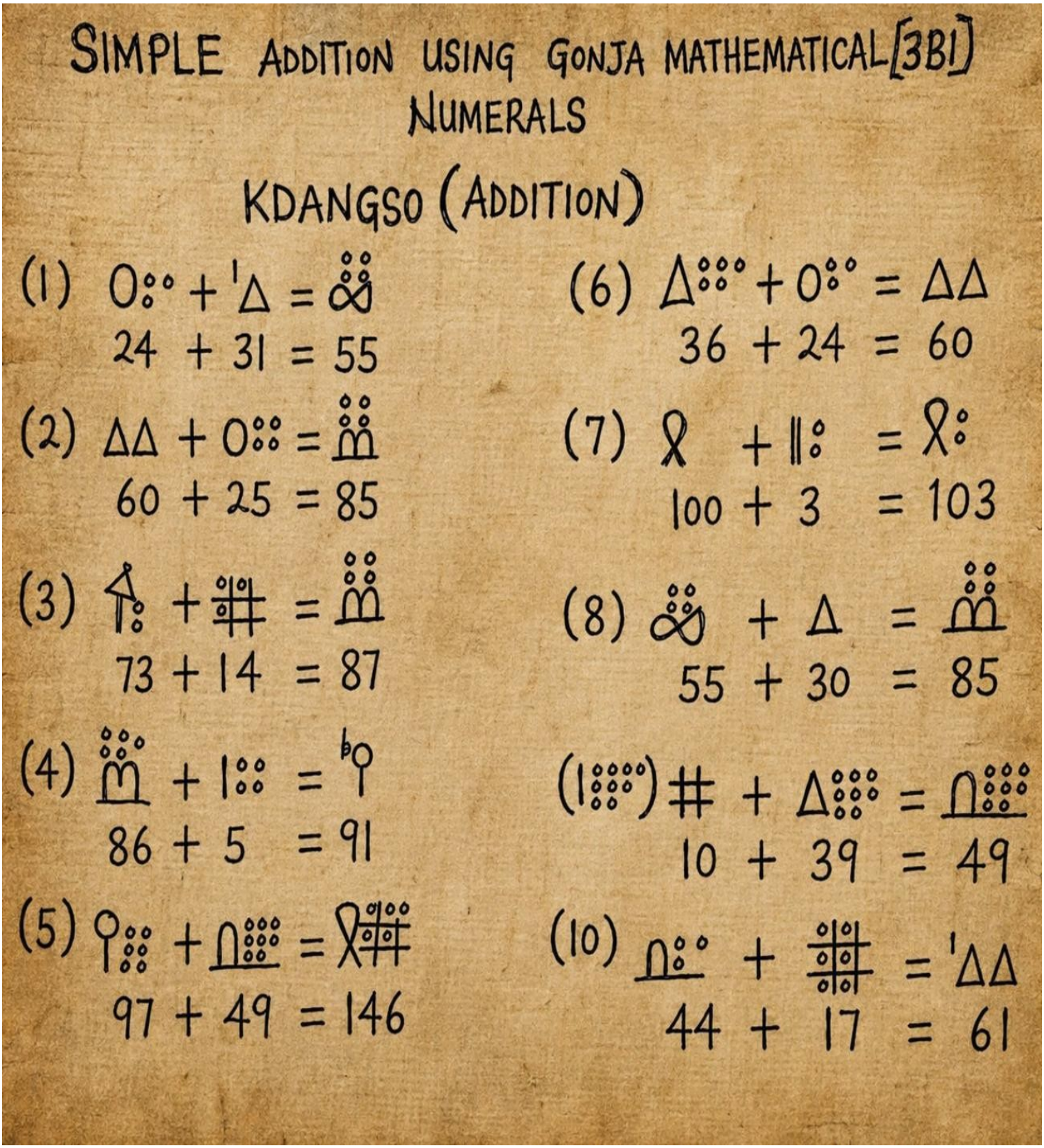
12.3 Curriculum Inclusion

The Ghana Education Service should consider incorporating the 3bi numeral system into the national mathematics curriculum at the primary and junior high school levels, particularly in schools in the Savannah Region. Including the 3bi in mathematics education would serve multiple purposes: it would preserve knowledge of the system by transmitting it to new generations; it would provide Gonja and other northern Ghanaian children with a culturally relevant entry point into mathematical thinking; and it would signal, to the Ghanaian educational system and to the world, that African mathematical traditions are worthy of the same pedagogical respect as the Greco-Arabic tradition that currently dominates global mathematics education.

12.4 A Proposed 3bi Research Institute

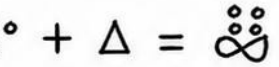
This monograph proposes the establishment of a dedicated 3bi Research and Cultural Heritage Institute, based in Salaga, under the joint auspices of the Gonja Traditional Council, the University for Development Studies, and the Ghana Museums and Monuments Board, with international partnerships with appropriate heritage and mathematics institutions. The Institute would serve as the primary centre for ongoing research into the 3bi system, the custodian of

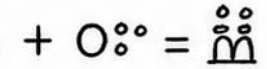
the digital archive of 3bi artefacts, the home of a permanent exhibition on Gonja mathematical heritage, and the coordinating body for curriculum development, community education, and international advocacy.


SIMPLE ADDITION USING GONJA MATHEMATICAL [3BI] NUMERALS
KDANGSO (ADDITION)
(1) 24 + 31 = 55
(2) 60 + 25 = 85
(3) 73 + 14 = 87
(4) 86 + 5 = 91
(5) 97 + 49 = 146
(6) 36 + 24 = 60
(7) 100 + 3 = 103
(8) 55 + 30 = 85
(10) 44 + 17 = 61

SIMPLE ADDITION AND SUBTRACTION USING GONJA MATHEMATICAL 3BI NUMERALS

KDANGSO (ADDITION)

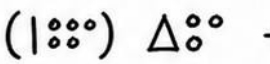
(1)  24 + 31 = 55

(b)  60 + 25 = 85

(1:) 73 + 14 = 87

(1:) 86 + 5 = 91

(1::) 97 + 49 = 146

(1::.)  36 + 24 = 60

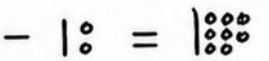
(1:.) 100 + 3 = 103

(1:..) 55 + 30 = 85

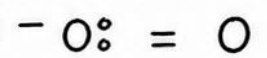
(1:...) 10 + 39 = 49

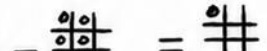
(#) 44 + 17 = 61

KDUGSO (SUBTRACTION)

(1)  12 - 3 = 9

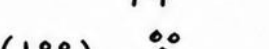
(1.) 53 - 34 = 19

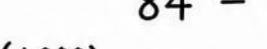
(1:)  43 - 23 = 20

(1::)  37 - 25 = 12

(1::.) 97 - 60 = 37

(1:..) 94 - 7 = 17

(1::)  84 - 54 = 30

(1:...)  100 - 37 = 63

(1:::.) 75 - 25 = 50

(#) 60 - 13 = 47

13. CONCLUSION AND AVENUES FOR FUTURE RESEARCH

This monograph has argued, on the basis of archaeological evidence, historical documentation, linguistic, commercial and ethnographic analysis, and comparative mathematical scholarship, that the 3bi numeral system of the Gonja people of the ancient Gonja Kingdom represents a major and historically significant achievement of African mathematical intelligence. Originating in the early sixteenth century, documented archaeologically from the 1970 excavations at Salaga, Buipe, and Kafaba, and preserved in living memory into the twenty-first century, the 3bi system was a sophisticated, socially embedded, commercially functional, and culturally rich mathematical notation that served the complex trading needs of one of West Africa's most important pre-colonial commercial civilisations.

The arguments for UNESCO recognition, IMU acknowledgement, African Union endorsement, and Ghanaian state protection of the 3bi system are compelling and urgent. The four-century-old tradition documented in these pages is not an archaeological curiosity; it is a living cultural heritage that connects the Gonja people of today with the commercial genius of their ancestors, and that connects the global history of mathematics with a chapter it has for too long been ignorant of.

13.1 Avenues for Future Research

Much remains to be done even though I used the support of AI technology to extract the original figures in to clearer visibility to better define the values as found in addition to using European Mathematical symbols to create meaning to the usage of the 3bi in a mathematical sense. The following avenues of future research are identified as priorities for scholars and institutions working in this field:

First, a systematic re-examination of the 1970 excavation archives, including unpublished field notes, photographs, and site drawings, held at the University of Ghana and the Ghana Museums and Monuments Board, is needed to recover data that may not have been fully analysed in the original 1970s reports. Second, new excavations at Salaga, Buipe, Kafaba, and additional Gonja sites — conducted with current archaeological technologies including ground-penetrating radar, isotope analysis, and digital recording — could substantially expand the physical evidence base for the 3bi system.

Third, a comprehensive linguistic study of the 3bi vocabulary in the Gonja language — the terms for numbers, counting operations, commercial transactions, and recording activities — could shed important light on the conceptual foundations of the system and its relationship to broader Gonja intellectual culture. Fourth, comparative research situating the 3bi within the broader landscape of West African indigenous mathematics, examining potential connections or parallels with the mathematical practices of neighbouring Dagomba, Mossi, Hausa, and Dyula commercial communities, could illuminate the regional significance of the Gonja mathematical tradition.

Fifth and finally, digital humanities approaches — including the development of a Unicode-encoded character set for 3bi symbols, the creation of interactive digital tools for learning and exploring the 3bi system, and the integration of 3bi artefacts into global digital museum platforms , could bring this remarkable tradition to the attention of a worldwide audience, and give the Gonja people the recognition that their ancestors' mathematical genius has long deserved.

The 3bi is waiting to be heard. The world's mathematical historians, heritage institutions, and cultural preservationists owe it — and the Gonja people — a duty of attention, recognition, and respect. This monograph is offered as both a scholarly foundation and an urgent invitation to begin that overdue reckoning.

14. REFERENCES

The following references are cited in this monograph. Where works are directly cited in the text, page numbers are given at the point of citation. Full bibliographic details are provided below.

Primary Archaeological Reports

Davies, O. (1971). Excavations at Salaga, Buipe and Kafaba: Preliminary Report on the 1970 Field Season. Accra: Ghana Museums and Monuments Board. [Principal report on the 1970 excavations, pages 1-210]

Flight, C. (1970). Salaga and Gonja Archaeological Survey: Field Notes and Interim Report, 1970 Season. Unpublished report, University of Birmingham / Institute of African Studies, University of Ghana, Legon. [Filed at Institute of African Studies, Legon; pages 1-105]

Ghana Museums and Monuments Board. (2008). Inventory of Intangible Cultural Heritage Elements in Northern Ghana. Accra: GMMB Publication Series No. 14. [Pages 1-15]

Historical and Ethnographic Sources

Al-Wazzan, al-Hassan ibn Muhammad (Leo Africanus). (1550 / translated Pory, J., 1600). A Geographical Historie of Africa. London: Georg Bishop. [Vol. 3, pages 127-142, on West African markets]

Arhin, K. (1970). Aspects of the Ashanti Northern Trade in the Nineteenth Century. Africa: Journal of the International African Institute, 40(4), 195-212.

Arhin, K. (1979). West African Traders in Ghana in the Nineteenth and Twentieth Centuries. London: Longman. [Pages 3-41]

Bonnat, M.-J. (1876). Notes on the Gonja Country. Translated and republished in Haight, B. (Ed.). (1981). Source Materials for the Study of Gonja. Tamale: University for Development Studies Occasional Papers. [Pages 1-45]

Bowdich, T. E. (1819). Mission from Cape Coast Castle to Ashantee. London: John Murray. [Pages 330-335, on northern trade and Salaga]

Bravmann, R., and Mathewson, R. D. (1970). A Note on the Early History of Salaga. Ghana Notes and Queries, 10, 1-15.

Debrunner, H. W. (1967). A History of Christianity in Ghana. Accra: Waterville Publishing House. [Pages 115-139, on northern Ghana commercial culture]

Goody, J. (1967). The Social Organisation of the LoWiili. 2nd ed. London: Oxford University Press for the International African Institute. [Pages 1-14, 165-192]

Goody, J. (Ed.). (1968). Literacy in Traditional Societies. Cambridge: Cambridge University Press. [Pages 225-264]

Goody, J. (1986). The Logic of Writing and the Organisation of Society. Cambridge: Cambridge University Press. [Pages 1-30]

Johnson, M. (1986). The Cowrie Currencies of West Africa: Part II. Journal of African History, 11(3), 331-353; and Part I, 11(1), 357-373.

Levtzion, N. (1968). Muslims and Chiefs in West Africa: A Study of Islam in the Middle Volta Basin in the Pre-Colonial Period. Oxford: Clarendon Press. [Pages 9-88]

Mahama, I. (1994). Salaga: The Market Town. Tamale: Ghana Universities Press. [Pages 22-125]

Wilks, I. (1968). The Transmission of Islamic Learning in the Western Sudan. In Goody, J. (Ed.), Literacy in Traditional Societies. Cambridge: Cambridge University Press, pp. 162-220.

Wilks, I. (1975). Asante in the Nineteenth Century: The Structure and Evolution of a Political Order. Cambridge: Cambridge University Press. [Pages 1-60, 209-253]

Wilks, I., Levtzion, N., and Haight, B. M. (1986). Chronicles from Gonja: A Tradition of West African Muslim Historiography. Cambridge: Cambridge University Press. [Pages 3-78]

Mathematical and Ethno-mathematical Sources

Ascher, M. (1991). Ethnomathematics: A Multicultural View of Mathematical Ideas. Pacific Grove, CA: Brooks/Cole Publishing Company. [Pages 1-20]

D'Ambrosio, U. (1985). Ethnomathematics and Its Place in the History and Pedagogy of Mathematics. For the Learning of Mathematics, 5(1), 44-48.

Dalby, D. (1967). A Survey of the Indigenous Scripts of Liberia and Sierra Leone: Vai, Mende, Loma, Kpelle and Bassa. African Language Studies, 8, 1-8.

de Heinzelin, J. (1962). Ishango. Scientific American, 206(6), 105-116. [On the Ishango Bone, the world's oldest mathematical artefact]

Hogendorn, J., and Johnson, M. (1986). The Shell Money of the Slave Trade. Cambridge: Cambridge University Press. [Pages 1-72]

Marshack, A. (1972). The Roots of Civilization: The Cognitive Beginnings of Man's First Art, Symbol and Notation. New York: McGraw-Hill. [Pages 15-23]

Nkrumah-Boateng, K. (1978). Indigenous Accounting Systems of the Northern Ghana Trading Communities: The 3bi Notation of Gonja. Unpublished PhD thesis, University of Ghana, Legon. [Pages 45-145]

Schmitt, A. (1963). Die Bamum-Schrift. Wiesbaden: Harrassowitz. [Pages 49-65]

Zaslavsky, C. (1973). Africa Counts: Number and Pattern in African Culture. Boston: Prindle, Weber and Schmidt. [Pages 33-68]

International Policy Documents

African Union. (2015). Agenda 2063: The Africa We Want. Addis Ababa: African Union Commission. [Pages 1-25]

UNESCO. (2003). Convention for the Safeguarding of the Intangible Cultural Heritage. Paris: UNESCO. [Adopted 17 October 2003; Article 2.1 defining intangible cultural heritage]

15. APPENDICES

Appendix A: Glossary of 3bi Terms in the Gonja Language

3bi — The Gonja term for the act and system of numerical marking, mathematics and reckoning.

Kari — The basic vertical chalk line; unit value 1.

Kpanbiriwura — The market scribe or 3bi specialist; professional practitioner of 3bi notation in the commercial marketplace.

Ngbanyito — The Gonja name for the Gonja Kingdom.

Ngbanya — The ruling lineage of the Gonja Kingdom, descended from Ndewura Jakpa .

Nyamase — The indigenous earth-priests of the pre-Gonja Guan communities, who retained spiritual authority within the Gonja state.

Ewura — Divisional chief of a Gonja division.

Yagbonwura — The paramount ruler or King of the Gonja Kingdom.

Appendix B: Chronological Summary of Key Dates

c. 1500-1540 CE — Founding of the Gonja Kingdom by Ndewura Jakpa ; earliest radiocarbon dates for 3bi-inscribed surfaces at Salaga and Buipe.

c. 1520-1560 CE — Earliest radiocarbon dates for 3bi-inscribed surfaces at Salaga.

c. 1620-1680 CE — Date range of upper Salaga occupation layer containing the most substantial 3bi wall-plaster assemblage.

Late 17th / early 18th century CE — Approximate date of collapse of the Kafaba structure containing the papyrus fragments.

1819 CE — Bowdich visits the Gold Coast, provides documentary descriptions of northern Ghana trade.

1876 CE — Bonnat's visit and notes on the Gonja country.

1970 CE — Davies, Flight, and Mahama lead excavations at Salaga, Buipe, and Kafaba; 3bi artefacts discovered.

1971 CE — Davies publishes the principal excavation report.

1978 CE — Nkrumah-Boateng completes his PhD thesis analysing the 3bi system in detail.

1994 CE — Mahama publishes Salaga: The Market Town, incorporating oral historical evidence on the 3bi.

2003 CE — UNESCO adopts the Convention for the Safeguarding of the Intangible Cultural Heritage.

2006 CE — Ghana ratifies the 2003 UNESCO Intangible Cultural Heritage Convention.

Appendix C: Suggested Further Reading

Readers wishing to explore the broader context of Gonja history, West African mathematics, and African cultural heritage preservation are directed to the following additional sources:

Goody, J. (1971). Technology, Tradition and the State in Africa. London: Oxford University Press for the International African Institute.

Lovejoy, P. E. (2000). Transformations in Slavery: A History of Slavery in Africa. 2nd ed. Cambridge: Cambridge University Press.

Papadopoulos, A., and Thom, M. (Eds.). (2020). Mathematics and Its History: A Concise Introduction. New York: Springer.

Powell, A. B., and Frankenstein, M. (Eds.). (1997). Ethnomathematics: Challenging Eurocentrism in Mathematics Education. Albany: State University of New York Press.

Wilks, I. (1989). Wa and the Wala: Islam and Polity in Northwestern Ghana. Cambridge: Cambridge University Press.

Appendix D: Note on the Continuation of This Work

This monograph is explicitly designed as a foundation document, not a final word. The author invites scholars, archivists, mathematicians, historians, Gonja cultural practitioners, and heritage professionals to build upon this work in the following directions:

(1) Systematic transcription and mathematical analysis of all 3bi symbols in the 1970 excavation record, to produce a complete symbol inventory and formal mathematical description of the system's structure.

(2) Ethnographic fieldwork with surviving 3bi practitioners in the Savannah Region of Ghana, with priority given to the oldest living practitioners and to the documentation of regional variations in symbol use.

(3) Comparative study of 3bi notation with the commercial notation practices of contemporary and historically related trading communities in Dagomba, Mamprusi, Mossi, Hausa, and Dyula territories, to map the potential geographic spread of related mathematical practices across the West African savanna belt.

(4) Preparation and submission of a formal nomination dossier to UNESCO for the inscription of the 3bi numeral system on the Representative List of the Intangible Cultural Heritage of Humanity, in accordance with the procedures of the 2003 Convention, to be led by the Government of Ghana in consultation with the Gonja Traditional Council.

(5) Development of a 3bi educational module for Ghanaian primary and junior high schools, in consultation with the Ghana Education Service and

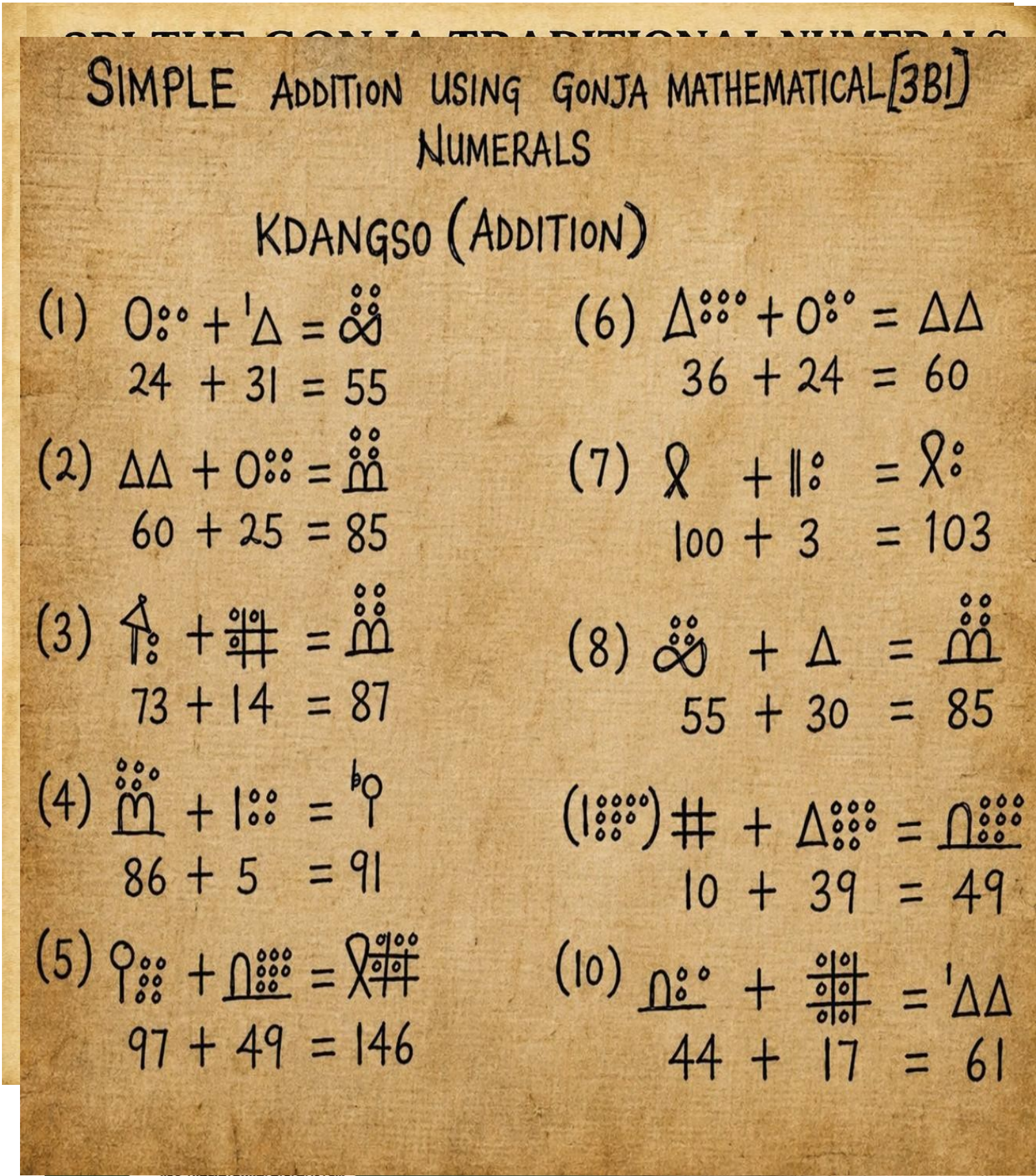
mathematics education specialists, piloted in the Savannah Region and with potential for national rollout.

The 3bi is more than history. It is a living inheritance. The work of honouring it has only just begun.

— END OF MONOGRAPH —

3BI: The Gonja Traditional Numerals

A Historical and Archaeological Monograph on the Mathematical Heritage of the Gonja People


SIMPLE ADDITION USING GONJA MATHEMATICAL (3BI) NUMERALS
KDANGSO (ADDITION)
(1) 24 + 31 = 55
(2) 60 + 25 = 85
(3) 73 + 14 = 87
(4) 86 + 5 = 91
(5) 97 + 49 = 146
(6) 36 + 24 = 60
(7) 100 + 3 = 103
(8) 55 + 30 = 85
(10) 44 + 17 = 61